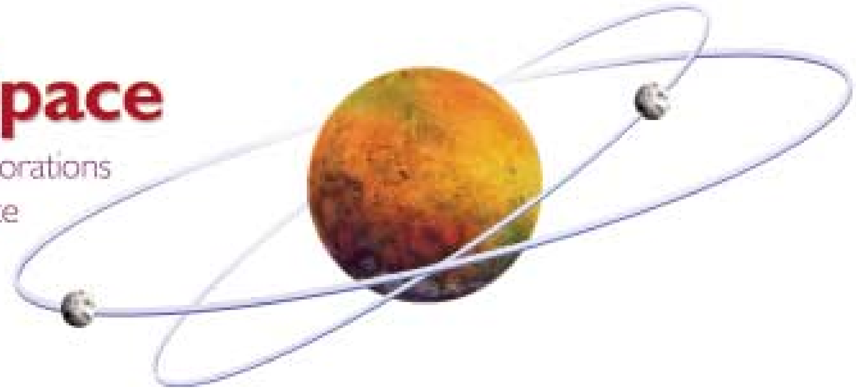
Geometry in Space



This technology-rich project uses explorations of Mars to teach geometry and science to middle and high school students.

National mathematics and science education standards emphasize that learning should be contextually situated, with students working together to solve realistic problems, using powerful modeling tools and methods. Although the research tools used by scientists may be unsuitable for use by most students, the fundamental concepts and procedures that characterize modeling and visualization may be learned using simpler versions of these tools. In doing so, students can formulate questions and conjectures.

The Geometry in Space Project, supported by grants from the Indiana Space Grant Consortium and Ball State University, sought to awaken an interest in the mathematics and science of modeling and remote sensing among middle school and high school mathematics and science teachers and students. In doing so, the authors and their undergraduate and graduate research assistants sought to promote and instill in this community a heightened sense of the value and timeliness of geometry in today's scientific world. This article focuses on the Web-based curricular resources created by the Geometry in Space project and the reactions of middle and high school students to these resources. (*Editor's note:* For the Geometry in Space URL and other Web addresses, see Resources at the end of this article.)

By David A. Thomas, John W. Emert, and Cynthia S. Thomas

Subject: Geometry, Mars exploration

Audience: Teachers, teacher educators

Grade Level: 6–12 (Ages 11–18)

Technology: Internet/Web, The Geometer's Sketchpad

Standards: *NETS•S* 3; *NETS•T* II (www.iste.org/standards); *NCTM* Connections, Geometry; Representation (http://standards.nctm.org/document); *NSES* Science Content Standards A, D (http://books.nap.edu/html/nses/html/index.html)

Investigations

The Geometry in Space curricular resources consist of a set of four ready-to-use, computer-based activities and a related slide presentation by NASA/JPL Mars Odyssey Mission chief scientist Dr. Steve Saunders. The titles of the investigations are:

- Orbital Mechanics: From Earth to Mars
- Finding a Place to Land
- Evaluating Your Landing Sites
- Mars in Perspective

In these activities, students learn important mathematical and scientific concepts related to the exploration of space and develop skill in the use of mathematical models and online scientific databases.

The Geometry in Space activities are interdisciplinary in nature, with connections to worthwhile mathematical, scientific, and technological concepts and questions, as seen in Table 1. In each activity, mathematics and science questions are investigated using computer- and Web-based modeling and information resources. Support for the use of technology resources in this manner is found in both the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000) and the *National Science Education Standards* (National Research Council [NRC], 1996). For instance, the mathematics standards state that students should learn to "recognize and apply mathematics in contexts outside of mathematics" (NCTM, p. 274) and "teachers can use simulations to give students experience with problem situations that are difficult to create without technology, or they can use data and resources from the Internet and Web to design student tasks" (NCTM, p. 26). Furthermore, the science standards state that "all students should develop abilities necessary to do science inquiry" (NRC, p. 110) and "the use of tools and techniques, including mathematics, … and the use of computers for the collection, summary, and display of evidence are part of this standard" (NRC, p. 145). The

Geometry in Space activities directly address these concerns in an engaging, exploratory manner.

Orbital Mechanics: From Earth to Mars. This activity focuses on the concept of planetary orbits and their mathematical features. Using Java-based modeling technologies and The Geometer's Sketchpad, students investigate the mathematical features of ellipses, note the elliptical features of planetary orbits, and compare the motions of the planets as they orbit the sun. For instance, the Kepler Ellipse Java applet (Figure 1) is used to illustrate the motion of a single planet along an elliptical orbit. In this model, students may observe both the shape of the orbit and variations in orbital velocity along the curve. By varying the shape of the ellipse, students may also observe the relationship between its shape and eccentricity, a numerical value displayed by the applet.

Having observed the general features of ellipses, students then investigate their underlying mathematical structures using The Geometer's Sketchpad (Figure 2). Using the model provided on the Geometry in Space Web site, students may vary the distance between the foci of the ellipse and other parameters associated with the curve, then observe the effect these changes have on the shape of the ellipse and the velocity of points tracing the curve. These observations add mathematical clarity to the intuitive impressions formulated using the Kepler Ellipse applet.

In the next stage of the investigation, students use the Visualize Solar System at a Given Epoch applet to observe the relative motions of the planets along their elliptical paths at any date in the past or future. Easily skipping from month to month or year to year, students quickly notice characteristic differences in the motions of the inner and outer planets relative to the mathematical features of their orbits.

With this introduction to the nature of elliptical curves and planetary motions, students then attempt what, for many, is the most challenging yet enjoyable feature of this activity—simulating a rocket launch that achieves a stable Earth orbit. This activity uses the demo version of the software package Orbit Xplorer. Through a trial and

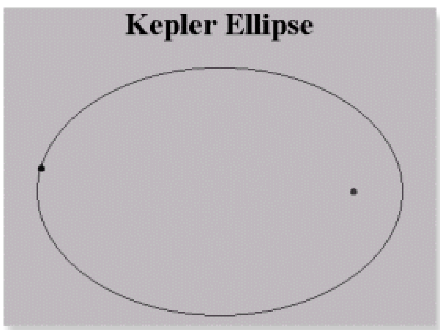


Figure 1. The Kepler Ellipse applet.

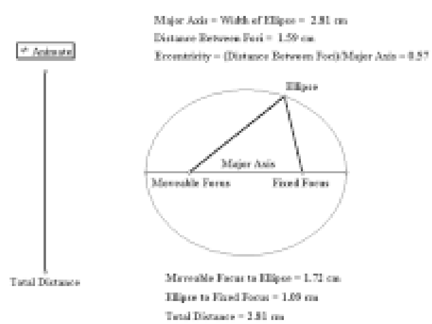


Figure 2. Mathematical structure of ellipses in The Geometer's Sketchpad.

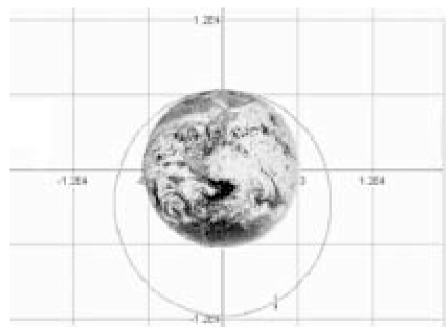


Figure 3. Achieving orbit with Orbit Xplorer.

Table 1. Concepts and Questions

Mathematics Content	Science Content	Facilitating Technologies	Interdisciplinary Connections
What is an ellipse?	What is an orbit?	Explore ellipses and orbits using The Geometer's Sketchpad and Java applets.	Planetary orbits are ellipses.
What is the relationship between a velocity vector and its vertical and horizontal components?	What velocity is required for a rocket to achieve Earth orbit?	Explore orbital mechanics using Orbit Xplorer.	Aim for the horizon: The horizontal component of velocity at launch is more important that the vertical component.
How do you measure objects in satellite images?	How large are landscape features (e.g., volcanos and canyons) on Mars compared to similar features on Earth?	NASA's Mars Explorer for the Armchair Astronaut and the Mars Global Surveyor Data Maps	Many of Mars's most prominent landscape features are far larger than corresponding features on Earth.
What is geometric perspective, and how is it used to create realistic views of three-dimensional objects on two-dimensonal computer screens?	What does the surface of Mars look like?	Explore 3-D modeling using The Geometer's Sketchpad, 3DEM, and NASA online data.	Students create realistic "fly-over" animations of a Martian volcano and canyon.

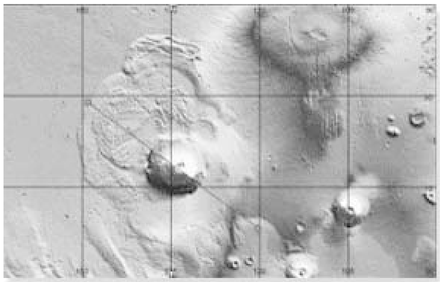


Figure 4. NASA's Mars Global Surveyor Data Map of Olympus Mons.

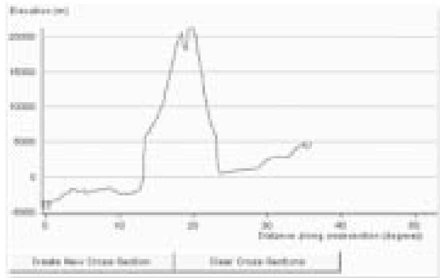


Figure 5. Elevation data of Olympus Mons.

Figure 6. Geological data of Olympus Mons.

error process, students vary the vertical and horizontal components of launch velocity until the model indicates that orbit has been achieved (Figure 3). In school presentations involving hundreds of middle and high school students, normally in groups of 20 to 30, this discovery has rarely been met with anything less than cheering and eager sharing of launch information.

The first activity concludes with a discussion of the nature of transfer orbits (also ellipses) from Earth to Mars. Using an online registration form, students may then add their names to a CD-ROM that will travel to Mars aboard a NASA spacecraft in 2003 and print a personalized certificate of registration.

Finding a Place to Land. The second activity makes use of a variety of online data resources to introduce students to Mars. Using the Planetary Data Systems' (PDS) Mars Explorer for the Armchair Astronaut Web page, students zoom in on prominent Martian features such as the giant volcano Olympus Mons and Valles Marineris, the Martian "grand canyon." Because the images encountered in this phase of the investigation are photographs, the students get a sense for the variety of landscape features found on Mars. In the first stage of this investigation, students use scaling information to get a sense for the size of these features.

In the second stage of the investigation, students explore the terrain of Mars using NASA's Mars Global Surveyor Data Maps Web page. These maps provide both elevation and geological data for the surface of Mars. For instance, in Figure 4, a line was drawn diagonally across Olympus Mons. The elevation changes along that line are graphed automatically in the browser window (Figure 5). Although the base of the volcano is below the Martian baseline elevation, it rises to a height of more than 20,000 meters. This is more than twice the height of Mt. Everest.

Figure 6 shows variations in geological data around Olympus Mons. Using this information, students may observe the variety of rocks and minerals on the surface (color coded by type) and their relationships to various geological structures.

As students use these resources to explore the surface of Mars, they begin to develop a sense for the shape and scale of its principal landscape features.

Evaluating Your Landing Sites. In the third activity, students learn more about the geology and mineralogy of various landing sites and review weather data gathered over several years. They use the geometry of longitude and latitude to specify locations on Mars and research related topographic, meteorologic, and geologic data available at selected sites. These data are used to extend their discussion of potential landing sites.

Mars in Perspective. The fourth activity focuses on the nature of geometric perspective and the creation of realistic three-dimensional (3-D) images and "fly-bys" of planetary surfaces from radar-generated elevation data. Students are introduced to fundamentals and history of geometric perspective using applets created with JavaSketchpad, The Geometer's Sketchpad, and the online MacTutor History of Mathematics Archives at St. Andrews University, Scotland. The remainder of the investigation is a tutorial on the use of the 3-D terrain rendering program 3DEM.

3DEM is a powerful tool for constructing realistic images (Figure 7) and movies based on digital elevation model data available from the U.S. Geological Survey, NASA, and other sources. Though this description may lead you to assume that 3DEM is difficult to use, nothing could be further from the truth. Within minutes of starting this investigation, middle school students can "fly" over the surface of Mars. The sense of personal engagement with Mars through this technology is powerful, indeed.

Assessment

The Geometry in Space Project has conducted eight workshops for about 400 teachers and students at the middle and high school levels. On each occasion, students worked in teams, with two or three students per computer. Our experience has led us to believe that small teams of students are more apt to resolve basic technical difficulties without a teacher's assistance, test ideas more thoroughly, and explain their findings more clearly than students working alone.

This phenomenon was particularly evident in the first activity as students sought to discover the velocity needed to attain Earth orbit. Essentially every student began this activity believing that the best way to get a rocket into

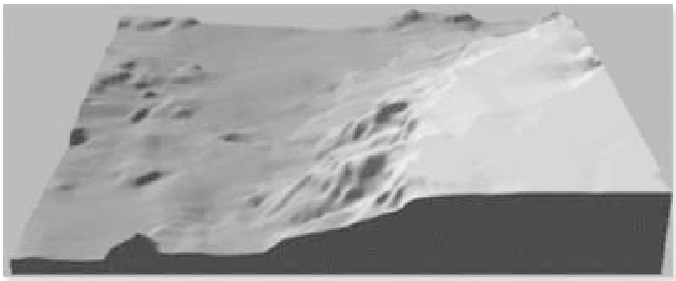


Figure 7. A 3DEM image of a canyon wall in Valles Marineris on Mars.

orbit was to aim straight up. Committed to this approach, the students repeatedly crashed their rockets. As they struggled with this result, we encouraged them to vary both the vertical and horizontal components of velocity. Eventually, one or more teams would begin focusing on the horizontal component of velocity at launch.

As first one team then another achieved orbit, the remaining teams first verified their solutions, then sought other solutions leading to different orbits. We would then challenge the students to discover the least velocity needed to achieve orbit. After meeting and mastering this challenge, students were prepared to formalize their findings using appropriate language and to clarify related mathematical, scientific, and technological issues.

A brief exit survey was conducted at the end of each workshop. Among other things, this survey asked the participants to rate the materials as being interesting, okay, or boring. In a sample of 209 responses, 74% of the students indicated that the materials were interesting, 22% rated the materials as okay, and 4% thought they were boring. The same students were also asked whether they would like to keep using the materials (61%), share the materials with a friend (35%), or do nothing (4%) with them. Overall, the reaction of the students to the Geometry in Space materials was unambiguous—they enjoyed using the materials and would like more experiences of the same kind. Teachers participating in the workshops also expressed similar reactions to the materials.

Students made the following comments on their survey response sheets:

- I enjoyed trying to orbit the earth with the rocket because it was fun to try different numbers to see what worked.
- I enjoy launching the rocket, because it was a challenge to see if you could orbit.
- Everything was interesting because I want to become a mathematical scientist when I am older.
- I liked flying over Mars because it looked like I was really at Mars.
- I liked everything about this program! It's awesome. You should make more of these!

We believe that the excitement and satisfaction we observed among the students and their teachers was based as much on their mastery of important mathematical and scientific questions as their engagement with the facilitating technologies. The technologies provided a vehicle of discovery, but the discoverers were the students, and they clearly savored that experience.

Resources

3DEM Software for Terrain Visualization and Flyby Animation: www.visualizationsoftware.com/3dem
Geometry in Space: www.cs.bsu.edu/homepages/dathomas/SpaceGrant/
MacTutor History of Mathematics Archives: http://www-history.mcs.st-and.ac.uk/history/
Mars Global Surveyor Data Maps: http://marsoweb.nas.nasa.gov/globalData/
Mars Explorer for the Armchair Astronaut: http://pdsimage.wr.usgs.gov/PDS/public/mapmaker/
Orbit Xplorer: www.ottisoft.com/orbit_x.htm
The Geometer's Sketchpad & JavaSketchpad: www.keypress.com/sketchpad

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National Research Council. (1996). *National science education standards.* Washington, DC: National Academy Press.

David A. Thomas is an associate professor of mathematics education in the Department of Mathematics at the University of Idaho, Moscow. In 1972, he was teaching mathematics, physics, and electronics at a high school in Oregon. His interest in calculator and computer technology developed as quickly as new tools became available. From the beginning, he and his students used them for the analysis of experimental data and to compute design parameters for several class projects.

Cynthia S. Thomas is an assistant professor of mathematics education in the Department of Teaching and Learning at Washington State University, Pullman. In 1972, as an elementary school teacher, she purchased her first calculator. During more than 30 years of teaching elementary and middle school, and now university students, she has been a vocal proponent of integrated uses of calculator and computer technologies in all classrooms.

John W. Emert is a professor of mathematics in the Department of Mathematical Sciences at Ball State University, Muncie, Indiana. In 1972, John was introduced to calculators by his junior high mathematics teacher, who set him to work building mathematical tables.

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